BAULKHAM HILLS HIGH SCHOOL

2017
YEAR 12 HALF YEARLY EXAMINATION

Mathematics Extension 1

General Instructions
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks – 70

Section I Pages 2 – 5

10 marks
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 10

60 marks
- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1 The point *P* divides the interval from $A(-1,-2)$ to $B(5,1)$ internally in the ratio 2:1. What are the coordinates of *P*?

(A) $\left(0,-\frac{3}{2}\right)$

(B) $(1,-1)$

(C) $\left(2,-\frac{1}{2}\right)$

(D) $(3,0)$

2 When $2x^3-3x^2+2a-4$ is divided by $x-1$ the remainder is -5. What is the value of *a*?

(A) 2

(B) 0

(C) –2

(D) –3

3 If $\cos x=\frac{3}{4}$ and $\sin x<0$, which of the following is the exact value of $\sin 2x$?

(A) $-\frac{3\sqrt{7}}{8}$

(B) $\frac{\sqrt{7}}{4}$

(C) $-\frac{\sqrt{7}}{4}$

(D) $\frac{3\sqrt{7}}{4}$

4 Which of the following is a point on the parabola $x^2 = 4ay$?

(A) $(0, a)$

(B) $(0, -a)$

(C) $\left(\frac{2a}{r}, \frac{a}{r^2}\right)$

(D) $(aq^2, 2aq)$

5 How many ways can 8 people be arranged around a circular table if Dineth must sit between Zhan and Xianyi?

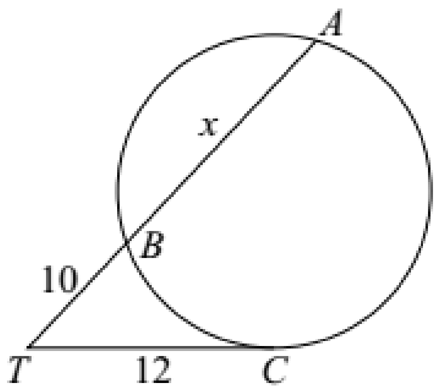
(A) 120

(B) 240

(C) 720

(D) 1440

6



In the diagram above, the tangent at *C* meets the secant *AB* at *T*. Given that *AB* = *x*, *BT* = 10 and *CT* = 12, the value of *x* is:

(A) 2

(B) $4\frac{2}{5}$

(C) 8

(D) $14\frac{2}{5}$

7 $\lim\limits_{x \to 0} \dfrac{\tan 3x}{2x}$ is equal to:

(A) 0

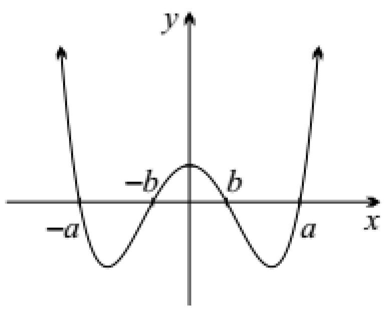
(B) $\dfrac{2}{3}$

(C) 1

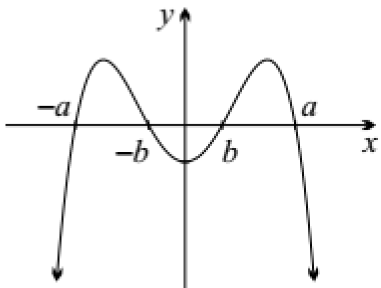
(D) $\dfrac{3}{2}$

8 Which diagram best represents $y = (x - a)^2(b^2 - x^2)$, where $a > b$?

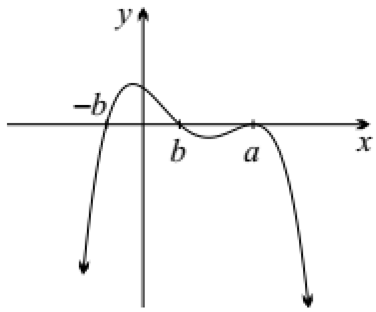
(A)



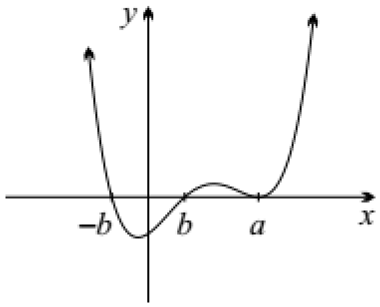
(B)



(C)



(D)



9 What is the value of $\cos^{-1}[\cos(3\pi + \alpha)]$ where α is an acute angle?

(A) α

(B) $\pi - \alpha$

(C) $\pi + \alpha$

(D) $3\pi + \alpha$

10 Given that the roots of $x^2 - 2x - 1 = 0$ are $\tan\alpha$ and $\tan\beta$, what is the value of $\alpha + \beta$?

(A) $\frac{\pi}{4}$

(B) $-\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $-\frac{\pi}{2}$

END OF SECTION I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour 45 minutes for this section

Answer each question on the appropriate answer sheet. Each answer sheet must show your NESA#. Extra paper is available.

In Questions 11 to 14, your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 11 ***(15 marks)*** Use a *separate* answer sheet

(a) Prove that $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta} = 2$ *2*

(b) Differentiate $\tan^{-1}\dfrac{5x}{4}$ *2*

(c) Find

(i) $\displaystyle\int \cos^2 x \; dx$ *2*

(ii) $\displaystyle\int \frac{dx}{\sqrt{1-2x^2}}$ *2*

(d) Express $8\cos x + 15\sin x$ in the form $R\cos(x-\alpha)$, giving α correct to the nearest degree. *2*

(e) Find the general solution to $2\sin x = \sqrt{3}$ *2*

(f) (i) How many nine letter arrangements can be made using the letters of the word; *1*

SCHOOLIES

(ii) In how many of the arrangements in part (i) do the vowels appear together? *1*

(iii) In how many of the arrangements in part (i) does the word **COOL** appear? *1*

Marks

Question 12 ***(15 marks)*** Use a *separate* answer sheet

(a) Find the greatest value of $\dfrac{\cos^2\left(\dfrac{\pi}{2}-\theta\right)-2\cos^2\theta}{24}$ for $0 \leq \theta \leq \dfrac{\pi}{2}$ *3*

(b) (i) Solve the inequality $\dfrac{x-5}{x-1} \leq -1$ *3.*

(ii) Hence, or otherwise, solve $\dfrac{\cos\alpha - 4}{\cos\alpha} \leq -1$, for $0 \leq x \leq \pi$ *2*

(c) In January 1995 the purebred dingo population on Fraser Island was 300. The population, *P*, since then can be modelled by;

$$P = 80 + Ae^{kt}$$

where *A* and *k* are constants, and *t* is the time since January 1995, in years.

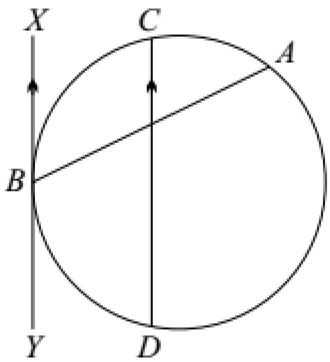
(i) Show that this model is a solution to the differential equation *1*

$$\frac{dP}{dt} = k(P-80)$$

(ii) In January 2015 it was found that the purebred population had dropped to 162. *2*
Show that purebred dingo population is decreasing at annual rate of approximately 5% per year.

(iii) Assuming this pattern continues, what will the purebred dingo population be in January 2050? *1*

(d) *3*



In the diagram, *AB* and *CD* are intersecting chords. The tangent at *B* is parallel to *CD*.

Copy this diagram into your answer booklet and prove that *AB* bisects $\angle CAD$

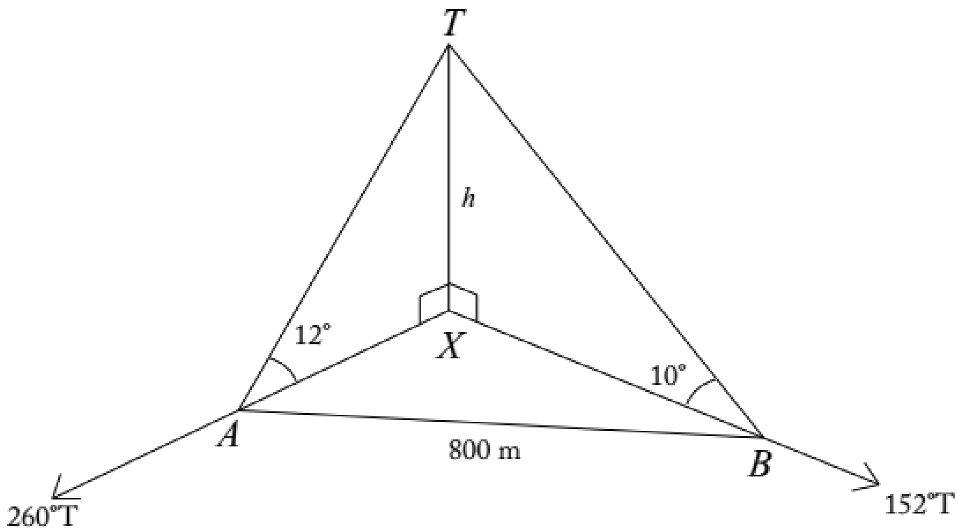
Marks

<u>Question 13</u> ***(15 marks)*** Use a *separate* answer sheet

(a) The radius of a circle is increasing such that the rate of increase of the area is $\pi^2 r$ cm^2/s. *2*

Calculate the rate of increase of the radius.

(b)



In the diagram above; *TX* represents a vertical tower of height *h* metres standing on the horizontal plane *AXB*.

Rachel and Marina are standing 800 metres apart on the same plane. Rachel is at point *A* on a bearing of $260°T$ from the tower and the angle of elevation to the top of the tower is $12°$. Marina is at point *B* on a bearing of $152°T$ from the tower and the angle of elevation to the top of the tower is $10°$.

(i) Explain why $\angle AXB = 108°$ *1*

(ii) Express *AX* in terms of *h* *1*

(iii) Find the height of the tower to the nearest metre *2*

(c) Use mathematical induction to prove that; *3*

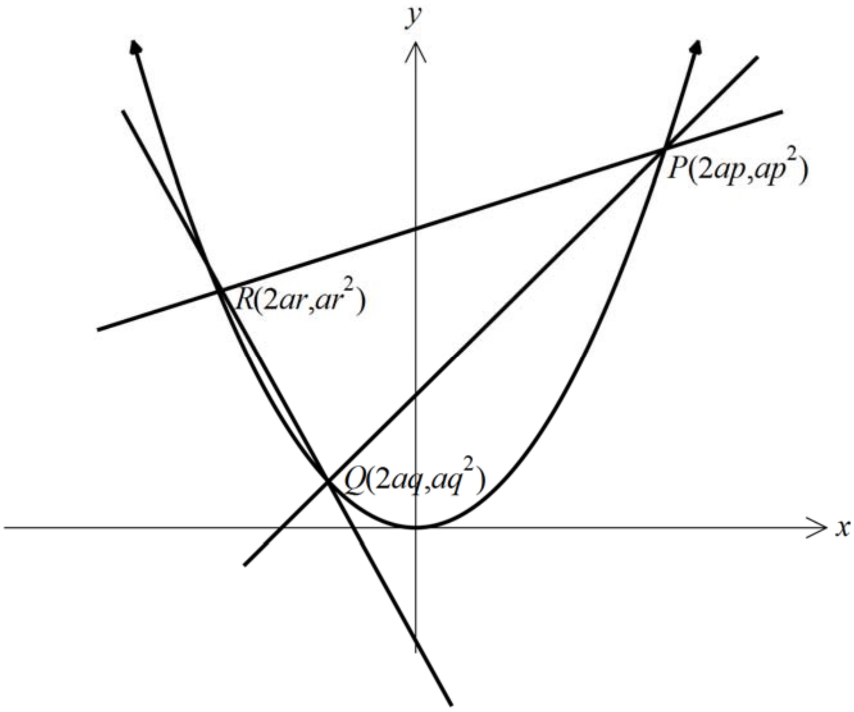
$$\sum_{r=1}^{n} \frac{5-4r}{5^r} = \frac{n}{5^n}$$

Question 13 continues on page 9

Marks

Question 13 ***(continued)***

(d) The diagram shows the parabola $x^2 = 4ay$. The distinct points $P(2ap, ap^2)$, $Q(2aq, aq^2)$ and $R(2ar, ar^2)$ lie on the parabola such that the normal to the parabola at *Q* and *R* both pass through the point *P*.



(i) Given that the equation of the normal at *Q* is $x + qy = aq^3 + 2aq$, show that $q^2 + pq + 2 = 0$ *2*

(ii) Show that the equation of the chord *QR* is given by $(q + r)x - 2y = 2aqr$ *2*

(iii) Show that *QR* always passes through the point $(0, -2a)$ *2*

End of Question 13

Marks

<u>Question 14</u> ***(15 marks)*** Use a *separate* answer sheet

(a) (i) State the domain and range of $y = 2\cos^{-1}(1-x)$ *2*

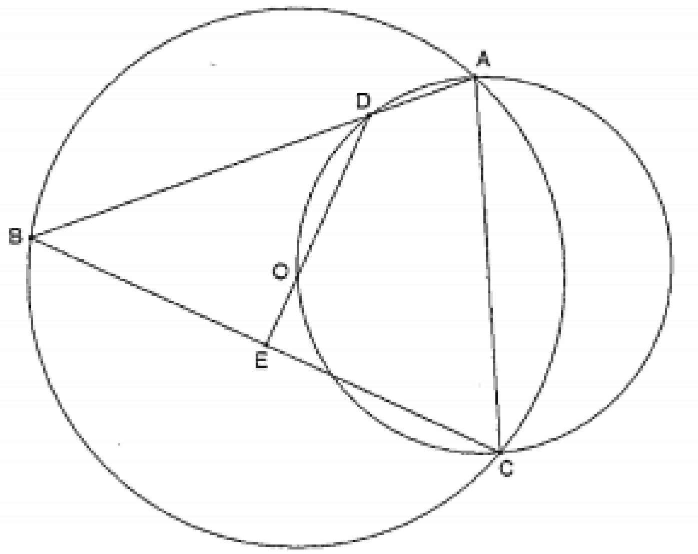
(ii) Sketch $y = 2\cos^{-1}(1-x)$ *1*

(iii) On the same set of axes as part (ii), sketch $y = -\pi x + 2\pi$ *1*

(iv) Explain why $\int_0^2 2\cos^{-1}(1-x)\ dx = \int_0^2 (-\pi x + 2\pi)\ dx$ *1*

(v) Without integrating, evaluate $\int_0^2 2\cos^{-1}(1-x)\ dx$ *1*

(b) *ABC* is a triangle inscribed in a circle with centre *O*. A second circle through the points *A*, *C*, *O* cuts *AB* at *D*. *DO* is produced to meet *BC* at *E*.



Copy the diagram into your answer booklet

(i) Prove that $\angle BOE = \angle BAC$ *3*

(ii) Prove that $BE = CE$ *2*

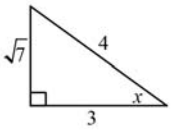
(c) (i) Use the factor theorem to show that $(a+b-c)$ is a factor of *2*

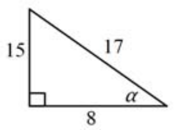
$$(a+b+c)^3 - 6(a+b+c)(a^2+b^2+c^2) + 8(a^3+b^3+c^3)$$

(ii) Hence factorise $(a+b+c)^3 - 6(a+b+c)(a^2+b^2+c^2) + 8(a^3+b^3+c^3)$ *2*

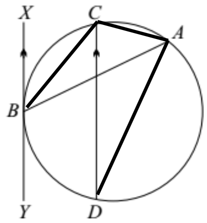
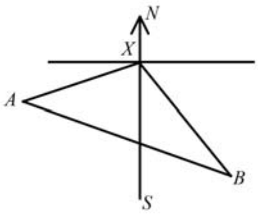
End of paper

BAULKHAM HILLS HIGH SCHOOL
YEAR 12 HALF YEARLY EXAMINATION 2017 SOLUTIONS

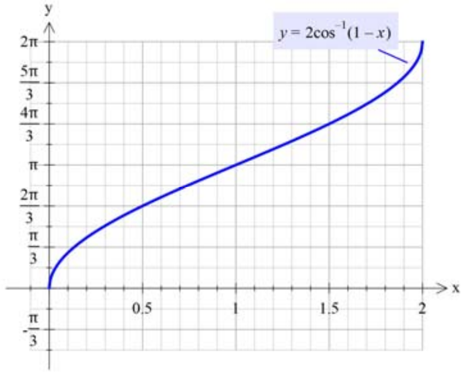
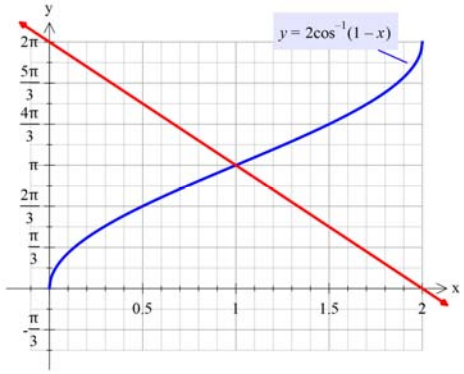
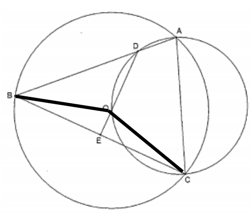
Solution	Marks	Comments
SECTION I		
1. D – $A(-1,-2)$ $B(5,1)$ $2:1$ $P=\left(\dfrac{1\times-1+2\times5}{2+1},\dfrac{1\times-2+2\times1}{2+1}\right)$ $=\left(\dfrac{9}{3},\dfrac{0}{3}\right)$ $=(3\ ,0)$	**1**	
2. B – $P(x)=2x^3-3x^2+2a-4$ $P(1)=-5$ $2-3+2a-4=-5$ $2a=0$ $a=0$	**1**	
3. A – $\sin 2x=2\sin x\cos x$ $=2\times-\dfrac{\sqrt{7}}{4}\times\dfrac{3}{4}$ $=-\dfrac{3\sqrt{7}}{8}$ 	**1**	
4. C – $x^2=\left(\dfrac{2a}{r}\right)^2$ $=\dfrac{4a^2}{r^2}$ $4ay=4a\times\dfrac{a}{r^2}$ $=\dfrac{4a^2}{r^2}$ $=x^2$	**1**	
5. B – Ways $=2!\times5!$ $=240$ Zhan and Xianyi must sit either side of Dineth = 2! Arrange group of three plus five other people Arrange 6 objects in a circle = 5!	**1**	
6. B - $AT\times BT=CT^2$ (square of tangent equals product of intercepts) $10(x+10)=12^2$ $10x+100=144$ $10x=44$ $x=\dfrac{22}{5}=4\tfrac{2}{5}$	**1**	
7. D - $\lim\limits_{x\to0}\dfrac{\tan 3x}{2x}=\lim\limits_{x\to0}\dfrac{\sin 3x}{3x}\times\dfrac{3}{2\cos 3x}$ $=\dfrac{3}{2}$	**1**	
8. C - $(x-a)^2\ \Rightarrow$ double root at $x=a$ $(b^2-x^2)=(b-x)(b+x)\ \Rightarrow$ single roots at $x=\pm b$ as $x\rightarrow\ =-\infty$ y behaves like leading term, $-x^3$	**1**	
9. B - $\cos(3\pi+\alpha)=\cos(\pi+\alpha)$ $=-\cos\alpha$ $\cos^{-1}[\cos(3\pi+\alpha)]=\cos^{-1}(-\cos\alpha)$ $=\pi-\cos^{-1}\cos\alpha$ $=\pi-\alpha$	**1**	
10. A - $\tan\alpha+\tan\beta=2$ & $\tan\alpha\tan\beta=-1$ $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ $=\dfrac{2}{1+1}$ $=1$ $\alpha+\beta=\dfrac{\pi}{4}$	**1**	

SECTION II		
Solution	**Marks**	**Comments**
QUESTION 11		
11(a) $\frac{\sin 3\theta}{\sin\theta}-\frac{\cos 3\theta}{\cos\theta}=\frac{\sin 3\theta\cos\theta-\cos 3\theta\sin\theta}{\sin\theta\cos\theta}$ $=\frac{\sin(3\theta-\theta)}{\frac{1}{2}\times 2\sin\theta\cos\theta}$ $=\frac{2\sin 2\theta}{\sin 2\theta}$ $=2$	**2**	**2 marks** • Correct solution **1 mark** • Correctly uses $\sin 2\theta$ result
11 (b) $\frac{d}{dx}\left(\tan^{-1}\frac{5x}{4}\right)=\frac{\frac{5}{4}}{1+\frac{25x^2}{16}}$ $=\frac{20}{16+25x^2}$ **OR** $\frac{d}{dx}\left(\tan^{-1}\frac{5x}{4}\right)=\frac{d}{dx}\left(\tan^{-1}\frac{x}{\frac{4}{5}}\right)$ $=\frac{\frac{4}{5}}{\frac{16}{25}+x^2}$ $=\frac{20}{16+25x^2}$	**2**	**2 marks** • Correct solution **1 mark** • Obtains a denominator of $16+25x^2$, or equivalent
11(c) (i) $\int \cos^2 x\,dx=\frac{1}{2}\int (1+\cos 2x)\ dx$ $=\frac{1}{2}\left(x+\frac{1}{2}\sin 2x\right)+c$	**2**	**2 marks** • Correct solution **1 mark** • Correctly uses $\cos 2\theta$ result
11(c) (ii) $\int \frac{dx}{\sqrt{1-2x^2}}=\frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\frac{1}{2}-x^2}}$ $=\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{x}{\frac{1}{\sqrt{2}}}\right)+c$ $=\frac{1}{\sqrt{2}}\sin^{-1}\sqrt{2}x+c$	**2**	**2 marks** • Correct solution **1 mark** • Uses correct standard integral
11(d)  $\alpha=\tan^{-1}\frac{15}{8}$ $=61.9275...^\circ$ $8\cos x+15\sin x=17\cos(x-62^\circ)$	**2**	**2 marks** • Correct solution **1 mark** • Correctly finds R or α
11 (e) $2\sin x=\sqrt{3}$ $\sin x=\frac{\sqrt{3}}{2}$ $x=\pi k+(-1)^k\sin^{-1}\frac{\sqrt{3}}{2}$ $x=\pi k+(-1)^k\left(\frac{\pi}{3}\right)$ where k is an integer	**2**	**2 marks** • Correct solution **1 mark** • establishes $\frac{\pi}{3}$ as the principal angle • uses the correct general angle formula
11(f) (i) Ways $=\frac{9!}{2!2!}$ $=90720$	**1**	**1 mark** • Answer may be left in factorial notation.
11(f) (ii) Ways $=\frac{4!}{2!}\times\frac{6!}{2!}$ $=4320$	**1**	**1 mark** • Answer may be left in factorial notation.
11(f) (iii) Ways $=1\times\frac{6!}{2!}$ $=360$	**1**	**1 mark** • Answer may be left in factorial notation.

QUESTION 12		
Solution	**Marks**	**Comments**
12 (a) $\dfrac{\cos^2\left(\frac{\pi}{2}-\theta\right)-2\cos^2\theta}{24}=\dfrac{\sin^2\theta-2\cos^2\theta}{24}$ $=\dfrac{1-3\cos^2\theta}{24}$ Now $0 \le \cos^2\theta \le 1$ Thus the greatest value of $\dfrac{\cos^2\left(\frac{\pi}{2}-\theta\right)-2\cos^2\theta}{24}$ is $\dfrac{1}{24}$	3	**3 marks** • Correct solution **2 marks** • Simplifies the fraction to a point where only one term is dependent upon θ • Finds the value of θ that gives a maximum **1 mark** • Attempts to simplify the expression by using a valid trig identity • Makes a valid attempt to find the value of θ that will give a maximum
12(b) (i) $\dfrac{x-5}{x-1} \le -1$ $x-1 \neq 0$ $x \neq 1$ $x-5=1-x$ $2x=6$ $x=3$  $1 < x \le 3$	3	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the two correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with the denominator. **0 marks** • Solves like a normal equation , with no consideration of the denominator.
12(b) (ii) $\dfrac{\cos\alpha-4}{\cos\alpha} \le -1$ let $\cos\alpha = u-1$ $\dfrac{u-5}{u-1} \le -1$ $1 < u \le 3$ $1 < \cos\alpha + 1 \le 3$ $0 < \cos\alpha \le 2$ $0 \le \alpha < \dfrac{\pi}{2}$	2	**2 marks** • Correct solution **1 mark** • Uses a valid substitution to transform the inequation into part (i) • Establishes the boundary values for the inequation.
12 (c) (i) $P = 80 + Ae^{kt}$ $\dfrac{dP}{dt} = Ake^{kt}$ $= k(80 + Ae^{kt} - 80)$ $= k(P-80)$	1	**1 mark** • Correct solution
12 (c) (ii) when $t=0$, $300 = 80 + Ae^0$ $= 80 + A$ $A = 220$ $t = 20$, $P = 162$ $162 = 80 + 220e^{20k}$ $e^{20k} = \dfrac{82}{220}$ $20k = \ln\left(\dfrac{41}{110}\right)$ $k = \dfrac{1}{20}\ln\left(\dfrac{41}{110}\right) = -0.0493.....$ $\therefore$ the population is decreasing at a rate of approximately 5% per year	2	**2 marks** • Correct solution **1 mark** • Establishes the value of A

Solution	Marks	Comments
12 (c) (iii) when $t = 55$, $P = 80 + 220e^{55k}$ $= 80 + 220\left(\frac{41}{110}\right)^{\frac{55}{20}}$ $= 94.5796...$ $\therefore$ in 2050 the population is predicted to be 95	1	**1 mark** • Correct solution *Note: no penalty for rounding error*
12 (d) Let $\angle XBC = \alpha$ $\angle BAC = \angle XBC = \alpha$ (alternate segment theorem) $\angle BCD = \angle XBC = \alpha$ (alternate $\angle$'s = , $XY \parallel CD$) $\angle BCD = \angle BAD = \alpha$ ($\angle$'s in the same segment) $\therefore \angle BAD = \angle BAC$ Thus AB bisects $\angle CAD$ 	3	**3 marks** • Correct proof **2 marks** • Establishes two or more angles that are equal to $\angle XBC$, or equivalent • Correct solution with poor reasoning **1 mark** • Uses a valid circle geometry theorem with correct reasoning
QUESTION 13		
13 (a) $\frac{dA}{dt} = \pi^2 r$ $A = \pi r^2$, $\frac{dA}{dr} = 2\pi r$ $\frac{dr}{dt} = \frac{dA}{dt} \times \frac{dr}{dA}$ $= \pi^2 r \times \frac{1}{2\pi r}$ $= \frac{\pi}{2}$ cm/s	2	**2 marks** • Correct solution **1 mark** • Uses the chain rule to combine two or more rates into a single expression
13 (b) (i) $\angle NXB = 152°$ & $\angle NXA_{reflex} = 260°$ $\angle BXS + \angle NXB = 180$ (straight $\angle NXS$) $\angle BXS + 152° = 180°$ $\angle BXS = 28°$ $\angle AXS + \angle NXS = \angle NXA_{reflex}$ (common $\angle$) $\angle AXS + 180° = 260°$ $\angle AXS = 80°$ $\angle AXB = \angle AXS + \angle BXS$ (common $\angle$) $\angle AXB = 80° + 28°$ $= 108°$ 	1	**1 mark** • Correct explanation *Note: formal geometrical proof not required, a simple explanation will suffice*
13 (b) (ii) $\frac{h}{AX} = \tan 12°$ $AX = \frac{h}{\tan 12°} = h\cot 12° = h\tan 78°$	1	**1 mark** • Correct answer
13 (b) (iii) $AX = h\tan 78°$, similarly $BX = h\tan 80°$ $AB^2 = AX^2 + BX^2 - 2AX \cdot BX \cdot \cos \angle AXB$ $800^2 = h^2\tan^2 78° + h^2\tan^2 80° - 2h^2\tan 78°\tan 80°\cos 108°$ $h^2 = \frac{800^2}{\tan^2 78° + \tan^2 80° - 2\tan 78°\tan 80°\cos 108°}$ $h = \frac{800}{\sqrt{\tan^2 78° + \tan^2 80° - 2\tan 78°\tan 80°\cos 108°}}$ $h = 95.08532019...$ $= 95$ metres (to the nearest metre)	2	**2 marks** • Correct solution **1 mark** • Uses the cosine rule in an attempt to find the height *Note: no penalty for rounding error*

<table>
<tr><th>Solution</th><th>Marks</th><th>Comments</th></tr>
<tr><td>
13 (c) When $n = 1$;
$LHS = \frac{5-4}{5} = \frac{1}{5}$ $\quad RHS = \frac{1}{5^1} = \frac{1}{5}$
$\therefore LHS = RHS$
Hence the result is true for $n = 1$
Assume the result is true for $n = k$
i.e. $\sum_{r=1}^{k} \frac{5-4r}{5^r} = \frac{k}{5^k}$
Prove the result is true for $n = k + 1$
i.e. Prove $\sum_{r=1}^{k+1} \frac{5-4r}{5^r} = \frac{k+1}{5^{k+1}}$
PROOF:
$\sum_{r=1}^{k+1} \frac{5-4r}{5^r} = \sum_{r=1}^{k} \frac{5-4r}{5^r} + \frac{5-4(k+1)}{5^{k+1}}$
$= \frac{k}{5^k} + \frac{1-4k}{5^{k+1}}$
$= \frac{5k+1-4k}{5^{k+1}}$
$= \frac{k+1}{5^{k+1}}$
Hence the result is true for $n = k + 1$,if it is true for $n = k$
Since the result is true for $n = 1$, then it is true for all positive integers by induction.
</td><td>3</td><td>
There are 4 key parts of the induction;
1. Proving the result true for $n = 1$
2. Clearly stating the assumption and what is to be proven
3. Using the assumption in the proof
4. Correctly proving the required statement
3 marks
• Successfully does all of the 4 key parts
2 marks
• Successfully does 3 of the 4 key parts
1 mark
• Successfully does 2 of the 4 key parts
</td></tr>
<tr><td>
13 (d) (i) P lies on the normal at Q
$(2ap, ap^2) \Rightarrow x + qy = aq^3 + 2aq$
$2ap + ap^2q = aq^3 + 2aq$
$q^3 - p^2q + 2q - 2p = 0$
$q(q^2 - p^2) + 2(q - p) = 0$
$q(q-p)(q+p) + 2(q-p) = 0$
$(q-p)[q(q+p) + 2] = 0$
$\therefore$ as $p \neq q$, $q^2 + pq + 2 = 0$
</td><td>2</td><td>
2 marks
• Correct solution
1 mark
• Substitutes P into the equation for the normal at Q
</td></tr>
<tr><td>
13 (d) (ii)
$m_{QR} = \frac{aq^2 - ar^2}{2aq - 2ar} = \frac{a(q+r)(q-r)}{2a(q-r)} = \frac{q+r}{2}$
$y - aq^2 = \frac{(q+r)}{2}(x - 2aq)$
$2y - 2aq^2 = (q+r)x - 2aq^2 - 2aqr$
$(q+r)x - 2y = 2aqr$
</td><td>2</td><td>
2 marks
• Correct proof
1 mark
• Find the slope of the chord QR
</td></tr>
<tr><td>
13 (d) (iii) when $x = 0, -2y = 2aqr$
$y = -aqr$
$q^2 + pq + 2 = 0$
similarly $r^2 + pr + 2 = 0$
Thus q and r are the roots of the quadratic $t^2 + pt + 2 = 0$
$\alpha\beta = 2$
$qr = 2$
$\therefore$ when $x = 0, y = -2a$ i.e. QR always passes through $(0, -2a)$
</td><td>2</td><td>
2 marks
• Correct proof
1 mark
• Shows $qr = 2$
• Identifies $(0, aqr)$ as the y-intercept
</td></tr>
</table>

QUESTION 14		
Solution	**Marks**	**Comments**
14 (a) (i) domain: $-1 \le 1-x \le 1$ $-2 \le -x \le 0$ $0 \le x \le 2$ range: $0 \le \frac{y}{2} \le \pi$ $0 \le y \le 2\pi$	**2**	**2 marks** • Correct answer **1 mark** • Finds either domain or range
14 (a) (ii) 	**1**	**1 mark** • Correct sketch
14 (a) (iii) 	**1**	**1 mark** • Correct sketch
14 (a) (iv) $y = 2\cos^{-1}(1-x)$ has rotational symmetry about the point $(1,\pi)$. Thus the area between $y = 2\cos^{-1}(1-x)$ and $y = -\pi x + 2\pi$ is the same from $x = 0$ to $x = 1$ as from $x = 1$ to $x = 2$	**1**	**1 mark** • Correct explanation involving symmetry
14 (a) (v) $\int_0^2 2\cos^{-1}(1-x)\,dx = \frac{1}{2} \times 2 \times 2\pi$ $= 2\pi$	**1**	**1 mark** • Correct solution
14 (b) (i) $\angle BOC = 2\angle BAC$ ($\angle$ at centre, twice $\angle$ at circumfernce on same arc) $\angle EOC = \angle BAC$ (exterior $\angle$, cyclic quad) $\angle BOC = \angle BOE + \angle EOC$ (common $\angle$) $\angle BOE = \angle BOC - \angle EOC$ $= 2\angle BAC - \angle BAC$ $= \angle BAC$ 	**3**	**3 marks** • Correct proof **2 marks** • Correct proof with poor reasoning • Uses two different valid circle geometry theorems with correct reasonong **1 mark** • Uses a valid circle geometry theorem with correct reasoning
14 (b) (ii) $OB = OC$ (= radii) $\angle BOE = \angle EOC$ (proven in (*i*)) OE is common $\therefore \Delta BOE \equiv \Delta COE$ (SAS) Thus $BE = CE$ (corresponding sides in $\equiv$ Δ's)	**2**	**2 marks** • Correct proof **1 mark** • Significant progress • Correct proof with poor reasoning

Solution	Marks	Comments
14 (c) (i) $P(a+b) = (a+b+c)^3 - 6(a+b+c)(a^2+b^2+c^2) + 8(a^3+b^3+c^3)$ If $(a+b-c)$ is a factor then $P(c) = 0$ $P(a+b) = [(a+b)+c]^3 - 6[(a+b)+c][(a+b)^2 - 2ab + c^2] + 8[(a+b)(a^2 - ab + b^2) + c^3]$ $= [(a+b)+c]^3 - 6[(a+b)+c][(a+b)^2 - 2ab + c^2] + 8[(a+b)[(a+b)^2 - 3ab] + c^3]$ $P(c) = [c+c]^3 - 6[c+c][(c^2 - 2ab + c^2] + 8[(c[c^2 - 3ab] + c^3]$ $= (2c)^3 - 12c(2c^2 - 2ab) + 8c(2c^3 - 3abc)$ $= 8c^3 - 24c^3 + 24abc + 16c^3 - 24abc$ $= 0$ $\therefore$ $(a+b-c)$ is a factor	2	**2 marks** • Correct solution **1 mark** • Demonstrates knowledge of the factor theorem
14 (c) (ii) If $(a+b-c)$ is a factor then so is $(a+c-b)$ and $(b+c-a)$ $\therefore (a+b+c)^3 - 6(a+b+c)(a^2+b^2+c^2) + 8(a^3+b^3+c^3) = k(a+b-c)(a+c-b)(b+c-a)$) Equating coefficients of a^3 $a^3 - 6a^3 + 8a^3 = -k\,a^3$ $k = -3$ $\therefore (a+b+c)^3 - 6(a+b+c)(a^2+b^2+c^2) + 8(a^3+b^3+c^3) = -3(a+b-c)(a+c-b)(b+c-b)$	2	**2 marks** • Correct proof **1 mark** • Finds another factor of the expression